1. (Widths of Massive Vector Bosons)

(a) If the vertex factor for the decay of a vector boson \( X \) into two spin-half fermions \( f_1 \) and \( \bar{f}_2 \) is 
\[ -ig_s\gamma^\mu(g_V - g_A\gamma^5)/2, \]
the shown that
\[ \Gamma(X \rightarrow f_1\bar{f}_2) = \frac{g_s^2}{48\pi}(g_V^2 + g_A^2)M_X, \]
where \( M_X \) is the mass of the boson and we have neglected the masses of the fermions.

(b) Assuming the Standard Model coupling, show that
\[ \Gamma(Z \rightarrow \nu_e\bar{\nu}_e) = \frac{g^2}{96\pi}\cos^2\theta_W M_Z. \]
Given that \( \sin^2\theta_W = 0.2310 \) and \( M_Z = 91.188 \text{ GeV} \), predict the numerical value of the partial width. Calculate the partial widths of the three decay modes \( Z^0 \rightarrow e^+e^-, u\bar{u}, \) and \( d\bar{d} \). Ignoring fermion masses, predict the total width of the \( Z \) in the Standard Model (don’t forget color).

(c) Repeat for the \( W^+ \rightarrow e^+\nu_e \) decay mode (take \( M_W = 80.423 \text{ GeV} \)). Calculate the partial widths of the two decay modes \( W^+ \rightarrow d\bar{u}, s\bar{u} \) (don’t forget to use CKM matrix elements [or Cabbibo angles]). Again, ignoring fermion masses, predict the total width of the \( W \) in the Standard Model.

2. (Forward-Backward Asymmetry when \( Z \) is included)
Calculate the differential angular distribution (in \( \cos \theta \)) for the process \( e^+e^- \rightarrow \ell^+\ell^- \) and find the forward-backward asymmetry, defined as:
\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \]
where \( \sigma_F (\sigma_B) \) is the cross-section for \( \ell^- \) to travel forward (backward) with respect to the \( e^- \) direction. Ignore the masses of the \( \ell^\pm \). Compare your answer with what is found in the Electroweak model and constraints on new physics section of the PDG on the web (pdg.lbl.gov).

3. (Making a Higgs directly)
Calculate the cross section for the reaction \( e^+e^- \rightarrow H \rightarrow f\bar{f} \), where \( f \) is a massive lepton.

4. (A toy symmetry breaking model)
Analyze the spontaneous breakdown of a global \( SU(2) \) symmetry. Consider the case of three real scalar fields, \( \phi_1, \phi_2, \phi_3 \), which constitute an \( SU(2) \) triplet (check past problem sets!), denoted:
\[ \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}. \]

The Lagrangian density is
\[ L = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi \cdot \phi), \]
\[ V = \frac{1}{2}\mu^2\phi \cdot \phi + \frac{1}{4}(\phi \cdot \phi)^2. \]
Assume that for $\mu^2 < 0$, the potential has a minimum at $\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$.

Then show that (a) the vacuum remains invariant under the action of the generator $T_3$, but not under $T_1$ or $T_2$; (b) the particles associated with $T_1$ and $T_2$ become massless (Goldstone) particles; and (c) the particle associated with $T_3$ acquires a mass of $\sqrt{-2\mu^2}$.