This final problem set should be treated as a take home exam, i.e., you are expected to work on it on your own, but can still use the resources of your notes, textbook(s), etc. Please contact me regarding questions of clarification. Hand in the completed exam at my office, dropping it in my mailbox (either in SW256 or SW117), e-mailing a scanned copy, or even FAXing to 812-855-2650. Good luck!

1. (Polarized τ’s, 25 pts.)
   (a) Derive the partial width for τ− → π−ντ. Compare your result to experimental data.
   (b) Derive the angular distribution of the π− relative to the τ− spin direction.
   (c) Derive the angular distribution of the e− in τ− → e−νeντ with respect to the τ spin. In the decay Z0 → τ+τ−, the τ’s are polarized. Which of the two decay modes above is the better analyzer of polarization?
   (d) Consider high-energy τ pair production in e+e− annihilation (s ≫ 4mτ2). What is the pion energy from τ→πν decay? (Ignore the pion mass for simplicity.) Now consider the case in which both τ’s decay via the τ→πν decay. If one π is observed with the maximum possible momentum, what is the energy spectrum of the other pion? (Hint: check out Halzen & Martin, section 6.6.)

2. (More τ’s and ντ neutrinos, 10 pts.)
   The leading candidate for producing τ leptons in hadronic interactions is hadronic production of Ds mesons followed by their decay Ds → τν. Given that the c¯s current couples weakly with approximately the same magnitude as the u¯d current, estimate the branching ratio for Ds → τν.

3. (Mixing it up, 20 pts.)
   (a) Draw the Feynman (box) diagrams responsible for K0– ¯K0, D0– ¯D0, B0d– ¯B0d, and B0s– ¯B0s mixing.
   (b) The predicted mass difference Δm between the mass eigenstates is proportional to the magnitude of the matrix element derived from the box diagrams. For K0– ¯K0 mixing for example, the box diagrams involving virtual quarks of flavor q and q′ with masses m_q and m_q′ lead to the prediction

   \[ Δm_K \approx \frac{G_F^2}{3\pi^2} f_K^2 |V_{qd}V_{qs}^*V_{q'd}V_{q's}^*| m_q m_{q'} m_K, \]

   where f_K ≈ 100 MeV, and the V_{ij} are the CKM matrix elements. Show that the dominant contribution to Δm_K comes from the box diagram containing two virtual charm quarks. Estimate Δm_K and compare with experiment.

   (c) Show that the dominant contributions to D0– ¯D0 and B0d– ¯B0d mixing come from the box diagrams containing virtual strange quarks and virtual top quarks, respectively. Obtain estimates of Δm_D and Δm_B. [Take f_K = f_D = f_B.] Explain why D0– ¯D0 mixing has not been (and is unlikely to be) observed. (Hint: convert Δm_D to a time and compare with the measured D0 lifetime of 4.1 ps.)

   (d) Draw Feynman diagrams for the decays B0d → π⁺π⁻ and ¯B0d → π⁺π−. Show that the
π⁺π⁻ final state in these decays is a CP eigenstate with CP = +1. Describe how you could measure CP violation with this mode.

4. (Even more neutrinos and mixing of a different kind, 20 pts.)
Supernova 1987A is located about 170,000 light years from Earth. The interactions of 10 neutrinos from the supernova were observed in a tank of 1000 tons of water within an interval of a second. The average neutrino energy was 10 MeV, and the energy varied from 5 to 20 MeV. The experimental signature of each neutrino interaction included recoil nucleons as well as a charged lepton.

(a) What are the weak interactions most likely involved in the creation and subsequent detection of the neutrinos?
(b) Estimate an upper limit on the neutrino mass from the observed data.
(c) Estimate the total energy in Joules that was liberated in the form of neutrinos during the supernova.
(d) Neutrinos of type $a$ created in the supernova could have transformed to another type $b$ while in transit, if both types of neutrino have nonzero mass. Derive an expression for the probability that neutrinos of type $a$ will appear to be type $a$ at the Earth in terms of the mixing parameter $\sin \theta = \langle \nu_2 | \nu_a \rangle$, where $\nu_1$ and $\nu_2$ are neutrino states of definite mass. Deduce a limit on the mass difference between neutrinos of types 1 and 2, supposing that the corresponding oscillation length is greater than the distance to the supernova.

5. (Higgs hunting, 15 pts.)
The dominant production mechanism for the Higgs boson in $e^+e^-$ collisions is via the Higgsstrahlung process $e^+e^- \rightarrow Z^0H^0$.

(a) Show that the cross section is given by
\[
\sigma = \frac{G_F^2 M_Z^4}{96 \pi} \left[1 + (1 - 4x_w)^2\right] \frac{8k}{\sqrt{s}} \left[\frac{k^2 + 3M_Z^2}{(s - M_Z^2)^2}\right],
\]
where $x_w = \sin^2 \theta_w$ and $k$ is the c.m. momentum of the Higgs (or produced $Z^0$). The process obviously proceeds with a virtual (or real) $Z$ boson as the propagator.
(b) Plot the cross section as a function of $\sqrt{s}$ for $m_H = 115$ GeV, 130, and 150 GeV. Estimate a relation for where the cross section peaks.

6. (Yes, you too can write science fiction..., 15 pts.)

(a) Explain how all four forces are involved in the formation and death of stars.
(b) In one of the numerous sequel books to 2001, A Space Odyssey, Arthur C. Clarke describes the “monoliths” of the original story (big black rectangles of unknown substance) as being self-assembling and replicating Turing machines. They drop down to Jupiter and transform the planet in such a way that it “ignites” in a fusion process and becomes a new star in our solar system. Describe two different ways that such an unlimited number of Turing machines could transform the content of Jupiter to allow fusion to begin. Assume that the monoliths don’t emit energy (i.e., just can’t heat everything up arbitrarily), but rather can transmute any element or particle into another element or particle.

I will return graded final problem sets to your mailboxes. Enjoy your summer! - Rick