1. (Hadron masses) As we discussed in class, a model for ground-state hadron masses can be
made in which a QCD analog of hyperfine splitting is used. In this scheme, meson masses are
given by:

\[ m(q_1 \bar{q}_2) = m_1 + m_2 + \frac{a(\sigma_1 \cdot \sigma_2)}{m_1 m_2}, \]

where \( m_1 \) and \( m_2 \) are the masses of the quarks and \( a \) is a positive constant.

(a) For the \( \pi \) (spin 0) and the \( K^* \) (spin 1), show that this expression gives

\[ m(\pi) = m_u + m_d - \frac{3a}{m_u m_d}; \quad m(K^*) = m_u + m_s + \frac{a}{m_u m_s}. \]

(b) Develop the analogous formula for spin-\( \frac{1}{2} \) and spin-\( \frac{3}{2} \) baryons.

(c) Obtain the value of \( a \) by looking at the \( \pi \) and \( K^* \) masses. Calculate the masses of all
the members of the 0\(^{-}\) and 1\(^{-}\) meson multiplets for charm and lighter using \( m_u = m_d = 0.310 \) GeV;
\( m_s = 0.48 \) GeV; \( m_c = 1.65 \) GeV. Compare your results with the meson
masses listed in the PDG.

(d) Check how well the predictions work with

\[ (\rho - \pi) = \frac{m_s}{m_u} (K^* - K) = \frac{m_c}{m_u} (D^* - D) = \frac{m_c m_s}{m_u^2} (D_{s}^* - D_s), \]

where the particle names are used to denote their masses.

2. (Yang-Mills non-Abelian theories) Derive the Yang-Mills Lagrangian for a scalar field theory
in which the three real scalar fields correspond to the triplet representation of \( SU(2) \). The
free-particle Lagrangian is

\[ \mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \phi)^2 - m^2 \phi^2 \right], \]

where

\[ \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \]