Introduction

Several weeks ago we found that we could synthesize complex sounds with a particular frequency, \( f \), by adding together sine waves from the harmonic series with a fundamental equal to the frequency \( (f, 2f, 3f, 4f, \ldots) \). We can reverse the process: a complex sound with particular frequency can be analyzed and quantified by the amplitude spectrum: the relative amplitudes of the harmonics.

Our goal today is to understand how the pitch, loudness, and timbre of a sound are represented in a spectrum. However, rather than use analog Band Filter, like last week, we will use the SoundScope program, which has powerful analysis software built in to calculate and graph the spectrum of a recorded sound.

Amplitude, loudness, and decibels

For the last couple weeks we have used the amplitudes to represent the different waves. We can represent these in a table (the choice for the amplitude of the fundamental wave to be \( 2V \) is completely arbitrary):

**Table 1A: The spectrum of a square wave**

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Another way to represent the relative amplitudes, which is used by SoundScope, is to graph the power for each harmonic. Since the sound power is proportional to the square of the amplitude for each harmonic, these numbers often get quite small for the higher harmonics. For this reason, the power is often expressed as a decibel. The power for each harmonic in decibels (dB) is:

\[
\text{Relative Power (dB)} = 10 \log_{10} [ (A_n/A_1)^2 ] = 20 \log_{10} [ (A_n/A_1) ]
\]

**Table 1B: The spectrum of a square wave**

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Since the logarithm of zero is negative infinity, the Relative Power (dB) for the even harmonics is "Error".

1) Try it: make up a table similar to the one above for the triangle wave you synthesized last week (Hint remember that the even harmonics had zero amplitude):

Table 2: The spectrum of a 440 Hz triangle wave:

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Power (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>440</td>
<td>0.0</td>
</tr>
<tr>
<td>880</td>
<td>6.0</td>
</tr>
<tr>
<td>1320</td>
<td>12.0</td>
</tr>
<tr>
<td>1760</td>
<td>18.0</td>
</tr>
<tr>
<td>2200</td>
<td>24.0</td>
</tr>
</tbody>
</table>

2) Now use the synthesizer to generate a triangle wave (use oscillator 1; change the function using the switches at the top left corner of the synthesizer box; switch other oscillators out). Turn the amplitude up so that it can just be observed. Sending too strong signal will result in a distorted signal due to saturation. The peak-to-peak amplitude recorded by the Sound Scope should not exceed 12.0V (see below)

a) Record the wave with SoundScope. The display on top shows the sound signal. The display on the bottom shows the Power Spectrum.

b) Notice that the power spectrum is made up of a series of peaks. Each peak represents one of the harmonics. You can use the mouse in the lower display to align the marker with the top of each peak. The numbers above the display show the frequency and the power (in decibels) for that frequency. Record the frequency and power for each peak in the table below. To obtain the last line of the table, you need to subtract the power level of the fundamental (P1) from each harmonic power level (Pn).
Table 3:Measured power spectrum for a triangle wave
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(c) How do your relative power numbers compare to what you calculated for Table 2?

d) Use the mouse to move the markers in the top part of the display in order to measure the peak-to-peak amplitude of the triangle wave. A reminder: it should be less than 12 V to avoid saturation:

_________________________ Volts
(If you forgot how to do this, consult the diagram on the bulletin board).

e) Now reduce the amplitude of the oscillator; record the sound again using SoundScope. Measure the peak-to-peak amplitude of the sound signal:

_________________________ Volts.

Record the power levels of all of the harmonics in the table below.

Table 4:Measured power spectrum for a triangle wave, new amplitude
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(f) Compared to Table 3:

Do all of the power levels change?
By how much do they change?
Do they change by different amounts or the same amount?
Do the relative power levels change?

g) The change in the power levels for each harmonics present should be the same as the change in the overall power level, which you can calculate from your measurements in part 3e (new amplitude) and 3d (the old amplitude):

\[
\text{Change in Power (dB)} = 10 \log_{10} \left[ \left( \frac{A_{\text{new}}}{A_{\text{old}}} \right)^2 \right]
\]

Are your measurements consistent with this observation?

4) Now switch in oscillator 4 (summing it to the triangle wave). Turn its amplitude halfway up of the oscillator 1 level (make this adjustment by recording signal for oscillator 4 first, before summing the two signals). Note that this harmonic was not present in the triangle wave signal

a) Record the sound signal. How do the sound signal and the power spectrum change?

b) Change the phase of the oscillator 4 by 180° and record the signal again. Does the sound change? Qualitatively, how do the sound signal and power spectrum change?

5) Now switch in oscillator 3. Adjust its amplitude (by recording the signal separately first) to be approximately half of that from oscillator 1. Note that this harmonic was present in the original triangle wave.

a) Record the sound signal. Qualitatively, how do the sound signal and the power spectrum change?
b) Change the phase of oscillator 3. Does the sound change? Record the signal again. Qualitatively, how do the sound signal and power spectrum change?

**Lesson:** If the Power Spectrum of the sound changes, you can probably hear the difference. If the Power Spectrum does not change, you can probably not hear the difference. For example, you should have found that changing the phase of the fourth harmonic did not change the power spectrum, but changing the phase of the third harmonic did matter. Why? The third harmonic was present in the original sound. Phase may matter when you add sounds that are close together (or the same) in frequency: see Synthesis -Adding Signals of the Same Frequency.

6) Switch Oscillator 1 back to a sine wave.

   a) Now switch in or out additional oscillators, one at a time, listen to the differences and record the signal after each switch. The sound should change every time, and new peaks should appear or peaks should disappear in the Power Spectrum. If you see no change, the amplitude for that oscillator is probably close to zero.

   b) Changing the phases of any of the oscillators should not change the sound or the power spectrum. The lesson is that the shape of the power spectrum determines the timbre.

7) We can now give very precise ways to define how to measure sounds made up of a harmonic series using the Power Spectrum:

   **Pitch:** the frequency of the fundamental of the harmonic series

   **Timbre:** the relative heights of the peaks in the power spectrum (compared to the fundamental)

   **Loudness:** the overall heights of the peaks in the power spectrum