4. (Griffiths 7.33) An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius $a$. The resulting electric field is given by:

$$\mathbf{E}(s, t) = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \log\left(\frac{a}{s}\right) \hat{z}$$

(a) Find the displacement current density $\mathbf{J}_d$.

(b) Integrate it to get the total displacement current, $I_d = \int \mathbf{J}_d dA$.

(c) Compare $I_d$ and $I$. (What’s their ratio?) If the outer cylinder were, say, 2 mm in diameter, how high would the frequency $\omega$ have to be for $I_d$ to be 1% of $I$? [This problem is designed to indicate why Faraday never discovered displacement currents, and why it is ordinarily safe to ignore them unless the frequency is extremely high.

5. (Griffiths 10.4) Suppose $V = 0$ and $\mathbf{A} = A_0 \sin(\kappa x - \omega t) \hat{y}$, where $A_0$, $\omega$, and $\kappa$ are constants. Find $\mathbf{E}$ and $\mathbf{B}$, and check that they satisfy Maxwell’s equation in vacuum. What conditions must you impose on $\omega$ and $\kappa$?

6. (Griffiths 10.6) Which of the potentials below are in the Coulomb gauge? Which are in the Lorentz gauge? (Note that these gauges are not mutually exclusive!)

(a) $V = 0$, $\mathbf{A} = \frac{\mu_0 \kappa}{4c} (ct - |x|)^2 \hat{z}$ for $|x| < ct$ and $=0$ otherwise.

(b) $V(\hat{r}, t) = 0$, $\mathbf{A}(\hat{r}, t) = \frac{1}{4\pi \varepsilon_0} \frac{qI}{r^2} \hat{r}$;

(c) $V = 0$, $\mathbf{A} = A_0 \sin(\kappa x - \omega t) \hat{y}$. 