**Formulae Sheet**

*Final Exam, Spring 2010*

**Conversion Factors:**
1 m = 3.281 ft; 1 in = 2.54 cm; 1 mile = 5280 ft; 1 mile = 1.61 km;
1 hr = 3600 s; 1 liter = 1000 cm³; 2π radians = 360°; 1 N = 1 kg · m/s²; 1 Hz = 1 cycle/s.

**Standard Prefixes:**

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giga</td>
<td>G</td>
<td>10⁹</td>
</tr>
<tr>
<td>Mega</td>
<td>M</td>
<td>10⁶</td>
</tr>
<tr>
<td>Kilo</td>
<td>k</td>
<td>10³</td>
</tr>
<tr>
<td>Centi</td>
<td>c</td>
<td>10⁻²</td>
</tr>
<tr>
<td>Milli</td>
<td>m</td>
<td>10⁻³</td>
</tr>
<tr>
<td>Micro</td>
<td>µ</td>
<td>10⁻⁶</td>
</tr>
<tr>
<td>Nano</td>
<td>n</td>
<td>10⁻⁹</td>
</tr>
</tbody>
</table>

Velocity, speed: \( v = \frac{\Delta x}{\Delta t} \); Average speed over time interval \( \Delta t \): \( v_{\text{ave}} = \frac{x_f - x_i}{t_f - t_i} \); Distance traveled: \( d = vt \).

Acceleration: \( a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \).

Density: \( \rho = \frac{m}{V} \).

Newton’s Second Law: \( F = ma \); Weight: \( W = mg \) \((g = 9.8 \text{ m/s}²)\).

Force due to a spring: \( F_{\text{spring}} = -k\Delta x \).

Pressure: \( P = \frac{F_A}{A} \).

Kinetic energy: \( KE = \frac{1}{2}mv^2 \); Potential energy due to gravity: \( PE_{\text{grav}} = mgh \), stored in spring: \( PE_{\text{spring}} = \frac{1}{2}kx^2 \).

Conservation of energy: \( KE_i + PE_i = KE_f + PE_f \).

Frequency, period: \( f = \frac{1}{T} \); Frequency mass/spring system: \( f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \); Frequency of pendulum: \( f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \).

Wavelength: \( \lambda = \frac{v}{f} \).

Speed of Sound:

(i) On a string or wire: \( v = \sqrt{\frac{T}{\mu}} \); 
(ii) In air: \( v = (331.3 + 0.6t) \text{ m/s, where } t \text{ is temperature in } °C \), i.e., at room temperature \((t = 20°C)\), \( v_{\text{sound}} = 343.3 \text{ m/s} \).

Importance of diffraction: if encountering an opening of approximate diameter \( d \), or an obstacle of typical size \( d \), diffraction is (i) important for \( \lambda \geq d \); (ii) much less important for \( \lambda \ll d \).

Interference pattern, two separated sources: constructive interference (“loud”) at angles from the perpendicular from two sources given by \( \sin \theta = \frac{m\lambda}{d} \), where \( m = 0, \pm 1, \pm 2, \ldots \).

Change in frequency due to Doppler Effect:

(i) Source moving, observer stationary: \( f_{\text{obs}} = f_s \left( \frac{v_{\text{sound}}}{v_{\text{sound}} + v_s} \right) \), \((-\text{approaching, } +\text{ receding})\);
(ii) Observer moving, source stationary: \( f_{\text{obs}} = f_s \left( \frac{v_{\text{sound}} \pm v_{\text{obs}}}{v_{\text{sound}}} \right) \), \((+\text{approaching, } -\text{ receding})\);

Velocity of wave on a string or wire of tension \( T \) and mass per unit length \( \mu = m/L \): \( v_{\text{wave}} = \sqrt{\frac{T}{\mu}} \).

Harmonic series for standing waves on a string of length \( L \) and mass per unit length \( \mu = m/L \):

\[ f_n = n f_1 = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}, \lambda_n = \frac{2L}{n}, \text{ where } n = 1, 2, 3, \ldots \]
Harmonic series for standing waves in a tube of length $L$ open at both ends:

$$f_n = nf_1 = \frac{nv}{2L}, \quad \lambda_n = \frac{2L}{n}$$

Harmonic series for standing waves in a tube of length $L$ open at one end, closed at other:

$$f_n = nf_1 = \frac{nv}{4L}, \quad \lambda_n = \frac{4L}{n}, \quad n = 1, 3, 5, \ldots, \text{ i.e., } n \text{ odd.}$$

Beats: two tones $f_A$ and $f_B$ presented simultaneously, beat frequency $f_{\text{beat}} = |f_B - f_A|$ is the frequency of the resultant amplitude modulation at a sum frequency of $f_{\text{sum}} = \frac{(f_A + f_B)}{2}$.

Interference pattern, two separated sources: constructive interference (“loud”) at angles from the perpendicular from two sources given by $\sin \theta = \frac{m\lambda}{d}$, where $m = 0, \pm 1, \pm 2, \ldots$.

Fourier Synthesis and Analysis:

Any complex periodic wave with fundamental frequency $f_1$ can be built up using Fourier synthesis from an infinite harmonic series (i.e., $f_n = nf_1$) of sinusoidal waves of different amplitudes, i.e., $A_n \sin(2\pi f_n t)$. Some standard or well-known waves are:

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Fourier Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>$A_1 = \text{constant, all other } A_n = 0$</td>
</tr>
<tr>
<td>Sawtooth (Ramp)</td>
<td>$A_n = \frac{A_1}{n}, \quad n = 1, 2, 3, \ldots$</td>
</tr>
<tr>
<td>Square</td>
<td>$A_n = \frac{A_1}{n}, \quad n = 1, 3, 5, \ldots$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$A_n = \frac{A_1}{n^2}, \quad n = 1, 3, 5, \ldots$</td>
</tr>
</tbody>
</table>

Sound Levels:

dB difference between two signals: $\text{dB} = 10 \log \frac{I_1}{I_2}$.

Sound Intensity Level ($L_i$ or $\text{SIL}$ in dB): $L_i = 10 \log \frac{I_i}{I_0}$, relative to a reference sound intensity $I_0 = 10^{-12} \text{ W/m}^2$.

Sound Pressure Level ($L_p$ or $\text{SPL}$ in dB): $L_p = 20 \log \frac{P_p}{P_0}$, relative to a reference sound pressure $P_0 = 2 \times 10^{-5} \text{ Pa}$ or $\text{N/m}^2$.

In most normal situations, $L_p = L_i$.

Sound Power Level (in dB) of a source: $L_W = 10 \log \frac{W}{W_0}$, relative to a reference sound power $W_0 = 10^{-12} \text{ W}$.

Inverting a logarithm: if $x = \log(y)$, then $y = 10^x$.

Definition of Intensity: $I = \frac{W}{A}$, where $W$ is power, and $A$ is area.

Variation of Intensity with Distance: $I = \frac{W}{4\pi r^2}$ where the source has sound power of $W$ Watts and the intensity is measured a distance of $r$ meters away.

Gain = $\frac{\text{Output Quantity}}{\text{Input Quantity}}$, Power Gain = $10 \log(W_0/W_i)$.

On an equal temperment scale (where one octave, i.e., a doubling or halving of frequency, is twelve semitones):

to move up from a note of frequency $f_0$ by $n$ semitones to frequency $f$:

$$f = f_0(1.0596)^n;$$

to move down $n$ semitones:

$$f = \frac{f_0}{1.0596^n}.$$
Ohm’s Law: \( V = IR \), or with impedance, \( Z \), \( V = IZ \).

Electrical Power: \( P = IV = I^2 Z = \frac{V^2}{Z} \).

Efficiency of Sound System = \( \frac{\text{Sound Power}}{\text{Electrical Power}} \times 100\% \).

Gain = \( \frac{\text{Output Quantity}}{\text{Input Quantity}} \), Power Gain = \( 10 \log(W_0/W_i) \).

Sensitivity of a microphone: \( S_V = \frac{V}{P} \); and in dBV, \( S_V(\text{dBV}) = 20 \log(V/P) \) where \( V \) is voltage in volts and \( P \) is pressure in Pa.

Focusing of a laser spot, area of smallest diffraction-limited spot \( \propto \lambda^2 \).

Nyquist Sampling Theory: \( f_{\text{sample}} \geq 2f_{\text{max}} \) where \( f_{\text{max}} \) is the maximum frequency to be digitized.

The largest decimal number that can be expressed with \( N \) bits is: \( 2^N - 1 \).

Dynamic Range (dB) = \( 10 \log(A_{\text{max}}^2/A_{\text{min}}^2) = 20 \log(A_{\text{max}}/A_{\text{min}}) \)

Direct/Early Sound:
\[
t = \frac{d}{v}. \quad t_1 = t_{\text{early}} - t_0 \quad \text{where} \quad t_0 = t_{\text{direct}} \quad \text{is the time of arrival of the direct sound (no bounces or relections) and} \quad t_{\text{early}} \quad \text{is the time of arrival of the first reflected sound.}
\]

Reverberant Time:
\[
RT = T_R = 0.161(V/A) \quad \text{where} \quad V \quad \text{is the volume, and} \quad A \quad \text{is the total absorption factor in sabins,} \quad A_{\text{tot}} = \sum S_i a_i, \quad \text{where} \quad S_i \quad \text{is the area of one component and} \quad a_i \quad \text{the absorption coefficient of that component.}
\]